Introduction to Robot Intelligence (CSCI-UA 480-072) Homework 5

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April 4, 2023

Submission Instructions

You must submit solutions to both the theory and coding portion of this homework to be eligible for full credit on this assignment.

Please navigate to the "Assignments" page of the course website (**linked here**) in order to download or copy the coding portion of the assignment.

You are strongly encouraged to typeset your answers to the theory questions below using LaTeX, via the course homework template (**linked here**). You must submit your answers to the coding problems by filling out the provided iPython notebook. We encourage you to use Google Colab to write and test your code.

This problem set is due on **April 13, 2023, 11:59 PM**. When you have completed both portions of the homework, submit them **on the course Gradescope as two separate files, with the coding portion in `.ipynb` format** by the due date. **No other forms of submissions will be accepted. Late submissions will also not be accepted.**

You may not discuss the questions in this problem set with other students.

Theory Questions

Problem 1: Numerical Dynamics

Each of the dynamics equations below relates acceleration to position, velocity, and controller torque u for some simple system. Suppose the state of each system is fully described by a position and velocity vector, $\mathbf{s}_t = \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}$.

Use first or second order Euler integration (i.e. $x_{k+1} = x_k + \dot{x}_k \cdot \Delta t$ and $x_{k+1} = x_k + \dot{x}_k \cdot \Delta t + \frac{1}{2}\ddot{x}_k \cdot (\Delta t)^2$, respectfully) to numerically re-write each dynamics equation, relating the system state $\mathbf{s}$ at some discrete timestep t_k to the state at the subsequent timestep, $t_{k+1} = t_k + \Delta t$. Are any of these dynamics equations linearizable about $\mathbf{s} = \mathbf{0}$?

1. $\ddot{x} = \cos^2(x) + \log(u) + 1$
2. $\ddot{x} = u\dot{x}^3 + x$
3. $\ddot{x} = \sin x + \sin u + u$

Problem 2: Understanding Linear Quadratic Regulators

1. Describe any limitations of LQR and its extensions. For which classes of control problems are LQR-based methods useful, and for which might we need to look to another family of methods?
2. Suppose that you are presented with a robot manipulator and asked to write a numerical LQR solver from scratch for a balancing problem with this manipulator. Describe all of the steps that you will need to take to accomplish this task.