Introduction to Robot Intelligence (CSCI-UA 480-072) Homework 4

Instructor: Lerrel Pinto

March 21, 2023

Submission Instructions

You must submit solutions to both the theory and coding portion of this homework to be eligible for full credit on this assignment.

Please navigate to the "Assignments" page of the course website (**linked here**) in order to download or copy the coding portion of the assignment.

You are strongly encouraged to typeset your answers to the theory questions below using LaTeX, via the course homework template (**linked here**). You must submit your answers to the coding problems by filling out the provided iPython notebook. We encourage you to use Google Colab to write and test your code.

This problem set is due on **March 30, 2023, 11:59 PM**. When you have completed both portions of the homework, submit them **on the course Gradescope as two separate files, with the coding portion in `.ipynb` format** by the due date. **No other forms of submissions will be accepted. Late submissions will also not be accepted.**

You may not discuss the questions in this problem set with other students.

Theory Questions

Question 1: Computing Jacobians

Derive the Jacobians of the functions below. Your derivations should be done *by hand*.

1. $g(x_1, x_2) = [x_1 x_2,\ \sin(x_1)]$
2. $f(x_1, x_2, x_3) = [\frac{x_1}{x_2},\ \cos^2(x_1 x_3)]$

Question 2: Existence of Jacobians

Under what conditions is a Jacobian for some function f well-defined, i.e. what conditions must f satisfy in order for its Jacobian to exist?

Question 3: Honey, I Hit a Singularity

Suppose we compute the Jacobian $\mathbf{J}$ of the kinematics equations for a manipulator end-effector with joint parameters $\mathbf{q} \in \mathbb{R}^n$, and we find that there exist valid configurations $\mathbf{q}'$ of our robot such that $\det(\mathbf{J}) = \mathbf{0}$ when $\mathbf{J}$ is evaluated at $\mathbf{q}'$.

1. How can we physically interpret this condition? What happens to the manipulator at the configurations $\mathbf{q}'$?
2. What happens to the numerical methods for inverse kinematics that we discussed in class (the Newton-Raphson method and the gradient descent-based approach) if an update yields joint parameters that have this property? Can either method recover from an update like this?