Introduction to Robot Intelligence (CSCI-UA 480-073) Homework 5

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Submission Instructions

This problem set is due on **Dec 3, 2024, 11:59 PM**.

You must submit solutions to both the theory and coding portions of this homework to be eligible for full credit on this assignment.

Please see the **Assignments page** of the course website for the coding portion of the assignment.

You are strongly encouraged to typeset your answers to the theory questions below using LaTeX, with the provided template (also on the Assignments page). You must submit your answers to the coding problems by filling out the provided IPython notebook. We encourage you to use Google Colab to write and test your code.

When you have completed both portions of the homework, submit them **as two separate files, with the coding portion in `.ipynb` format**. **No other forms of submissions will be accepted. Late submissions will also not be accepted.**

You may not discuss the questions in this problem set with other students.

Theory Questions

Section I: Jacobians

Question 1.1: Computing Jacobians

Derive the Jacobians of the function below. Your derivations should be done *by hand*.

$f(x_1, x_2, x_3) = [\frac{x_1}{x_2},\ \cos^2(x_1 x_3)]$

Question 1.2: Existence of Jacobians

Under what conditions is a Jacobian for some function f well-defined, i.e. what conditions must f satisfy in order for its Jacobian to exist?

Question 1.3: Honey, I Hit a Singularity

Suppose we compute the Jacobian $\mathbf{J}$ of the kinematics equations for a manipulator end-effector with joint parameters $\mathbf{q} \in \mathbb{R}^n$, and we find that there exist valid configurations $\mathbf{q}'$ of our robot such that $\det(\mathbf{J}) = \mathbf{0}$ when $\mathbf{J}$ is evaluated at $\mathbf{q}'$.

1. How can we physically interpret this condition? What happens to the manipulator at the configurations $\mathbf{q}'$?
2. What happens to the numerical methods for inverse kinematics (the Newton-Raphson method and the gradient descent-based approach) if an update yields joint parameters that have this property? Can either method recover from an update like this?

Section II: Dynamics & Control

Problem 2.1: Numerical Dynamics

Each of the dynamics equations below relates acceleration to position, velocity, and controller torque u for some simple system. Suppose the state of each system is fully described by a position and velocity vector, $\mathbf{s}_t = \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}$.

Use first or second order Euler integration (i.e. $x_{k+1} = x_k + \dot{x}_k \cdot \Delta t$ and $x_{k+1} = x_k + \dot{x}_k \cdot \Delta t + \frac{1}{2}\ddot{x}_k \cdot (\Delta t)^2$, respectfully) to numerically re-write each dynamics equation, relating the system state $\mathbf{s}$ at some discrete timestep t_k to the state at the subsequent timestep, $t_{k+1} = t_k + \Delta t$. Are any of these dynamics equations linearizable about $\mathbf{s} = \mathbf{0}$?

1. $\ddot{x} = \cos^2(x) + \log(u) + 1$
2. $\ddot{x} = u\dot{x}^3 + x$
3. $\ddot{x} = \sin x + \sin u + u$

Problem 2.2: Understanding Linear Quadratic Regulators

1. Describe any limitations of LQR and its extensions. For which classes of control problems are LQR-based methods useful, and for which might we need to look to another family of methods?

2. Suppose that you are presented with a robot manipulator and asked to write a numerical LQR solver from scratch for a balancing problem with this manipulator. Describe all of the steps that you will need to take to accomplish this task.